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Coordinate Transformations for CIS

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0.0 REFERENCE DOCUMENTS

[1] *Data Delivery Interface Document*, ESOC, CL-ESC-ID-0001, May 2000 (Version 3).

[2] *CIS Level_1 Science Data Timing*, I. Dandouras, CESR, March 1999.

[3] *Report of the Data Products Task Group for the Cluster Science Data System*, P.W. Daly et al., MPAE-Lindau, DS-MPA-TN-0001, June 1994.

[4] *Coordinate Transformations for Cluster*, M. Hapgood, JSOC, DS-JSO-TN-0008, June 1997.

[5] *Cluster Software Tools - Part I - Coordinate Transformations Library*, P. Robert, CRPE, DT/CRPE/1231, July 1993.

[6] C.T. Russell, *Geophysical Coordinate Transformations*, Cosmic Electrodynamics, 2, 184, 1971.

[7] *Mathematics Manual of Maglib Library*, J.C. Kosik, CNES, PLASMA-LO-MAGLIB-00161-CN, November 2001.

0.1 SCOPE

The CIS moments of the ion distribution functions, calculated onboard and supplied in the raw (Level_0) data, are given in an Instrument-Build coordinate system. During the data processing performed on ground, in order to generate the CSDS (Level_2) data files [*Ref. 3*], a series of coordinate transformations is performed. The end result is the transformation of the data from the Instrument-Build to the GSE coordinate system. This document describes these coordinate transformations.

0.2 INTRODUCTION

Coordinate transformation of a vector $\mathbf{V_i}$, in an initial coordinate system **i**, to a vector $\mathbf{V_f}$ to a final coordinate system **f**, is performed by the matrix multiplication:

$$\mathbf{V_f = M_{if} \bullet V_i}$$

where $\mathbf{M_{if}}$ is the 3 × 3 transformation matrix that rotates a vector from system **i** to system **f.** $\mathbf{M_{if}}$ is a function of the direction cosines of system **f** with respect to system **i**. Any coordinate transformation can be decomposed in a series of intermediate transformations, the resultant transformation matrix being the multiplication product of the intermediate transformation matrices.

More details on how the transformation matrices are defined can be found in *Ref. 4*.

For CIS, the transformation from the Instrument-Build to the GSE coordinate system is decomposed in 7 elementary transformations:

1. **Instrument-Build** to **SR2** :
   - 1a. **Instrument-Build** to **Attitude System**
   - 1b. **Attitude System** to **SR**
   - 1c. **SR** to **SR2**
2. **SR2** to **SR1**
3. **SR1** to $\mathbf{GEI_{J2000}}$
4. $\mathbf{GEI_{J2000}}$ to **GEI**
5. **GEI** to **GSE**

In the following these coordinate systems and the corresponding rotations are described.

1.0 Instrument-Build to SR2

1.a Instrument-Build to Attitude System

Figure 1.a.1 shows the **HIA** instrument-build reference frame (INS) with respect to the spacecraft-build attitude system (A), defined in *Ref. 1*. Note that the attitude system Z-axis is the spacecraft cylinder symmetry axis. The Sun direction shown is approximately the one at the start of the data acquisition cycle, for magnetospheric modes.

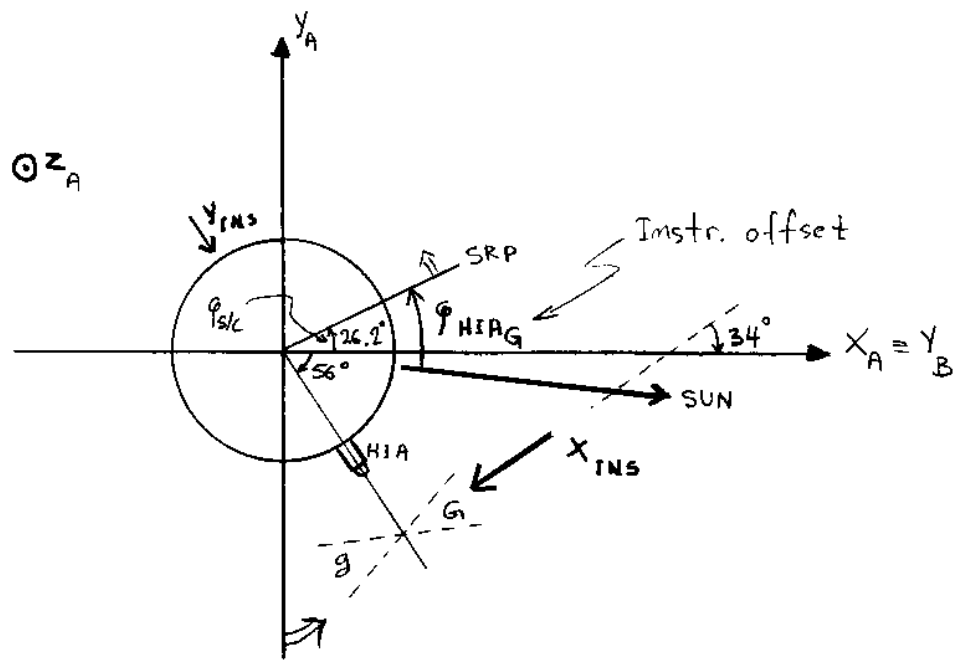

Fig. 1.a.1

X_{INS} in this figure corresponds to Magnetospheres Modes

The $\mathbf{M_{1a}}$ (instrument-build to attitude system) transformation matrix, for **HIA magnetospheric modes** data acquired from the High (G) geometric factor, thus is:

$$M1a := \begin{pmatrix} \cos(HIA\_to\_att) & \sin(HIA\_to\_att) & 0 \\ -\sin(HIA\_to\_att) & \cos(HIA\_to\_att) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where: **HIA_to_att = 180°-34° (magnetospheric modes: P2 telemetry product)**.

For **solar wind modes**, during which **HIA** data as telemetry product P4 are acquired from the low (g) geometric factor, the HIA instrument-build reference frame is rotated by 180° (opposite looking direction):

HIA_to_att = -34° (solar wind modes: P4 telemetry product).

Furthermore, the Instrument system Z-axis is reversed (opposite direction to the Z_A axis), resulting to a non-right handed system. The $\mathbf{M_{1a}}$ matrix thus becomes (cf. Figure 1.a.2):

$$M1a := \begin{pmatrix} \cos(\text{HIA\_to\_att}) & \sin(\text{HIA\_to\_att}) & 0 \\ -\sin(\text{HIA\_to\_att}) & \cos(\text{HIA\_to\_att}) & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

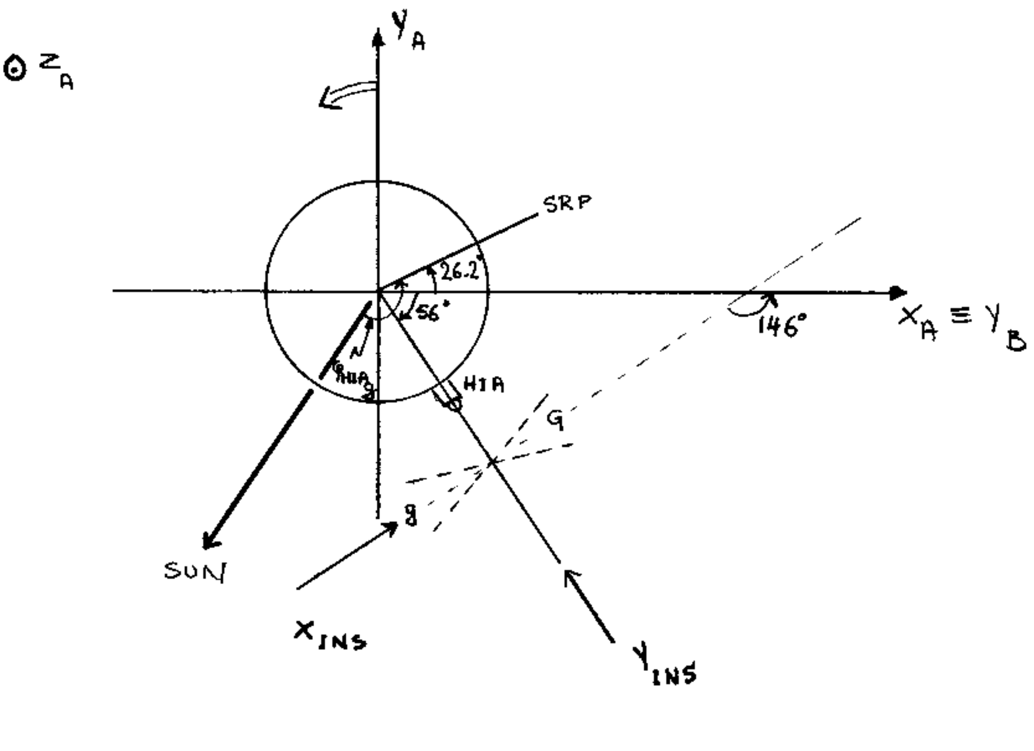

Fig. 1.a.2

X_{INS} in this figure corresponds to Solar Wind Modes

For **CODIF** the $\mathbf{M_{1a}}$ (instrument-build to attitude system) transformation matrix, for data acquired from the **High (G) geometric factor**, is (cf. Figure 1.a.3) :

$$M1a := \begin{pmatrix} \cos(\text{CODIF\_to\_att}) & \sin(\text{CODIF\_to\_att}) & 0 \\ -\sin(\text{CODIF\_to\_att}) & \cos(\text{CODIF\_to\_att}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where**: CODIF_to_att = -34° (High-G side)**.

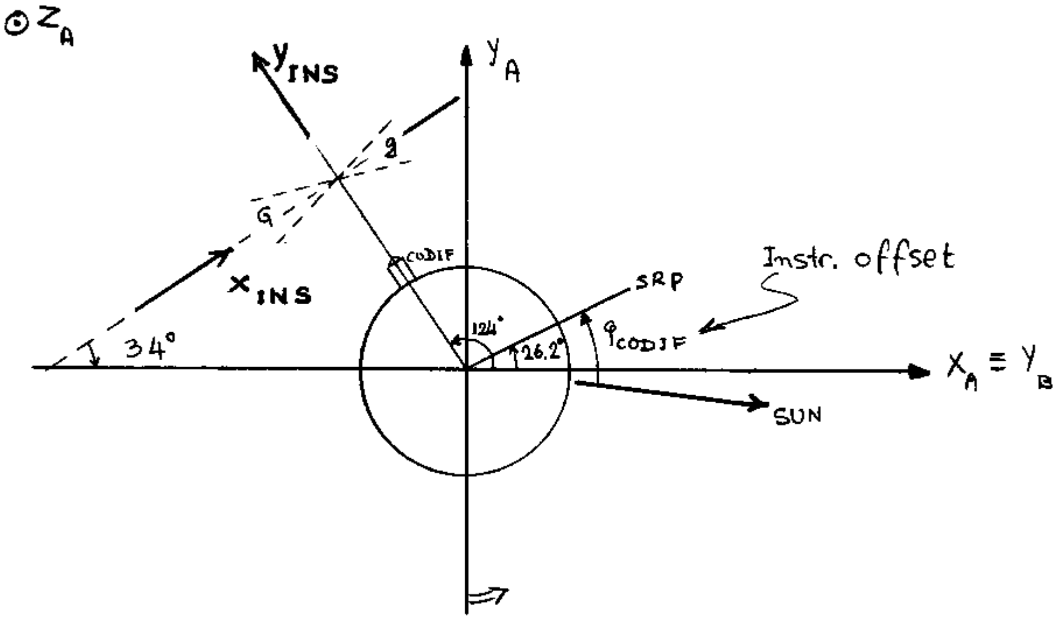

Fig. 1.a.3

X_{INS} in this figure corresponds to the High-G side of CODIF

For data acquired from the **low (g) geometric factor**, the CODIF instrument-build reference frame is rotated by 180° (opposite looking direction). In the $\mathbf{M_{1a}}$ matrix thus:

CODIF_to_att = 180°-34° (low-g side).

Furthermore, the Instrument system Z-axis is reversed (opposite direction to the Z_A axis), resulting to a non-right handed system. The $\mathbf{M_{1a}}$ matrix thus becomes:

$$M1a := \begin{pmatrix} \cos(\text{CODIF\_to\_att}) & \sin(\text{CODIF\_to\_att}) & 0 \\ -\sin(\text{CODIF\_to\_att}) & \cos(\text{CODIF\_to\_att}) & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

1.b Attitude System to SR

SR is the Spin Reference system, its Z-axis is aligned with the maximum principal inertia axis of the spacecraft, and it is shown in Figure 1.b.1.

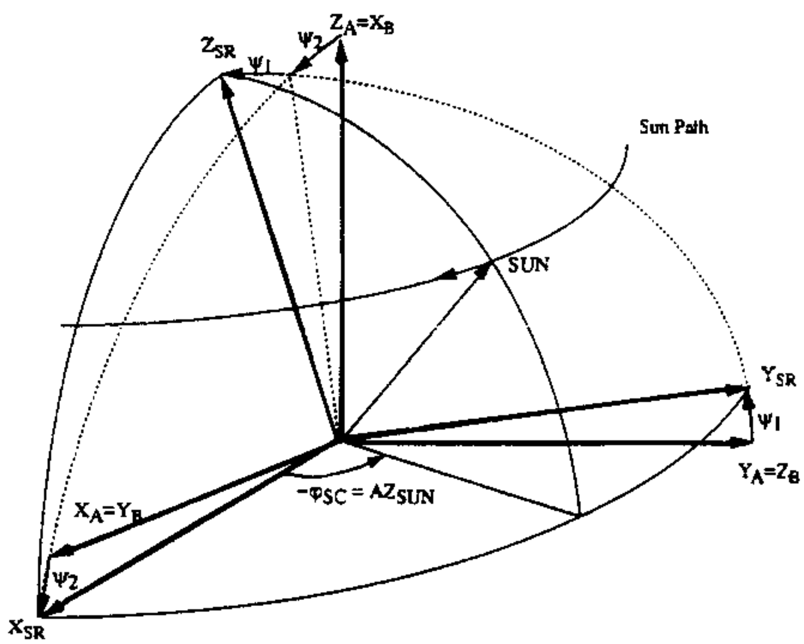

Fig. 1.b.1 (adapted from Ref. 1)

The angles ψ_1 and ψ_2 , that give the rotation from the spacecraft-build attitude system to the spin reference system, are normally very small (absolute_value < 0.1°) and are given in the spacecraft attitude file (`SATT`), supplied by ESOC (*Ref. 1*).

The $\mathbf{M_{1b}}$ (attitude to spin reference system) transformation matrix thus is:

$$M1b := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(TPSI1) & \sin(TPSI1) \\ 0 & -\sin(TPSI1) & \cos(TPSI1) \end{pmatrix} \cdot \begin{pmatrix} \cos(TPSI2) & 0 & -\sin(TPSI2) \\ 0 & 1 & 0 \\ \sin(TPSI2) & 0 & \cos(TPSI2) \end{pmatrix}$$

where `TPSI1` and `TPSI2` are the angles ψ_1 and ψ_2 respectively.

1.c SR to SR2

The SR system is a reference frame that is rotating with the spacecraft. The data representation in this system, defined above, is valid at the start of the instrument data acquisition cycle.

The SR2 system has the same Z-axis as the SR system, but is a non-rotating one. Its X-axis is defined as the intersection of the SR system X-Y plane with the SR meridian containing the Sun, i.e. the Sun Reference Pulse is generated when the Sun Sensor crosses this meridian (*Ref. 4*).

The transformation from the SR to the SR2 system is thus a rotation around the Z-axis by an angle

$$\phi = \phi_{s/c} - \phi_{INS}$$

where:

- $\phi_{s/c}$ is the spacecraft phase, i.e. the rotation angle of the half-plane defined by the $+Z_{SR}$ and $+X_{SR}$ axes, around the maximum principal axis of inertia ($+Z_{SR}$), from the time when the Sun direction was contained in this plane to the Sun Reference Pulse. $\phi_{s/c} = 360° -$ `SCPHAS`, where `SCPHAS` is given in the spacecraft attitude file (`SATT`), supplied by ESOC (*Ref. 1*). The expected value for $\boldsymbol{\phi_{s/c}}$ is **26.1°**.

- ϕ_{INS} is the instrument spin phase adjustment, i.e. the phase delay from the Sun Reference Pulse to the start of the instrument data acquisition cycle. Note that ϕ_{INS} can be different for CODIF and HIA, and that for HIA it is mode dependent, changing value when the instrument switches from a magnetospheric mode to a solar wind mode (or vice versa). The expected nominal values of $\boldsymbol{\phi_{INS}}$ are **26.37°** for **CODIF (all modes)** and for **HIA magnetospheric modes**, and **149.76°** for **HIA solar wind modes**. *Ref. 2* gives how ϕ_{INS} is calculated from the CIS telemetry data.

The $\mathbf{M_{1c}}$ (SR to SR2) transformation matrix thus is:

$$M1c := \begin{pmatrix} \cos(PHIsc - PHIins) & \sin(PHIsc - PHIins) & 0 \\ -\sin(PHIsc - PHIins) & \cos(PHIsc - PHIins) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where PHIsc is $\phi_{s/c}$ and PHIins is ϕ_{INS} .

2. SR2 to SR1

The SR1 system (*Ref. 4*) has the same Z-axis as the SR system. Its X-axis is defined as the intersection of the SR2 (or SR) system X-Y plane with the X-Y plane of the GEI (Geocentric Equatorial Inertial) system (Fig. 2.1).

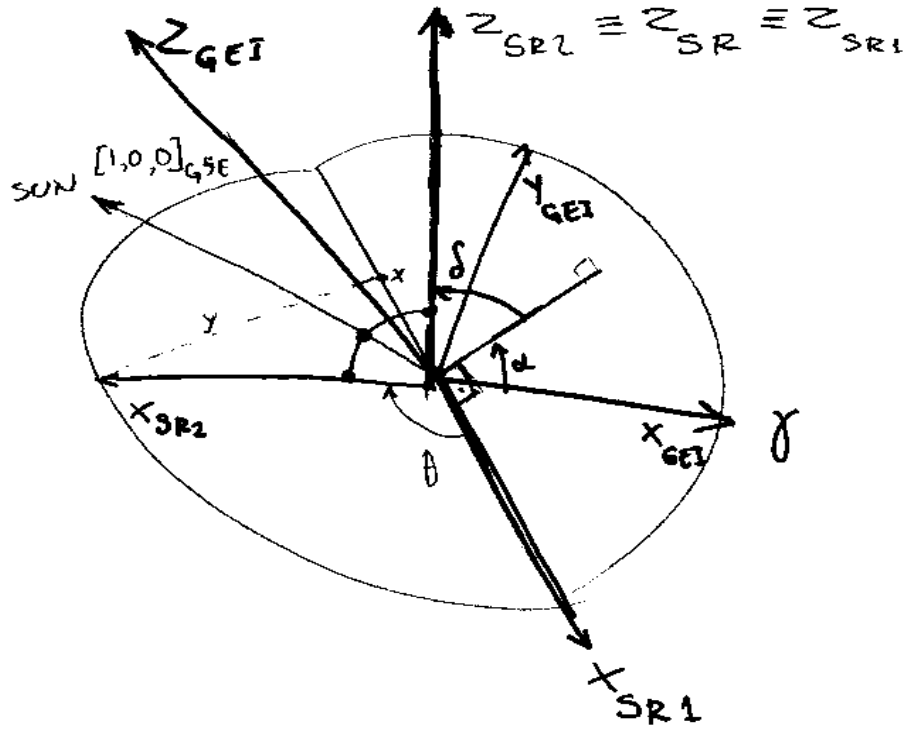

Fig. 2.1

GEI system with respect to SR2

The transformation matrix $\mathbf{M_2}$ from the SR2 to the SR1 system is thus a rotation around the Z-axis by an angle -θ:

$$M2 := \begin{pmatrix} \cos(\text{theta}) & -\sin(\text{theta}) & 0 \\ \sin(\text{theta}) & \cos(\text{theta}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

To calculate θ, we take into account the fact that the X_{SR2} axis is defined as the intersection of the SR system X-Y plane with the SR meridian containing the Sun, i.e. containing the GSE direction {1,0,0}. Thus, if we convert this vector to the SR1 system, we can derive the longitude of the Sun in the SR1 system. This longitude is identical to the required angle θ.

If $\mathbf{T_{xyz}}$ = {x,y,z} is the unit vector giving the Sun direction in the SR1 system, then

$$Txyz := M3inv \cdot Pinv \cdot T2inv \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

where:

- **T2inv** is the GSE to GEI transformation matrix. This is the inverse of the GEI to GSE matrix, defined in section 5.
- **Pinv** is the GEI to GEI_{J2000} (the GEI system corresponding to year 2000) transformation matrix. This is the inverse of the GEI_{J2000} to GEI matrix **P**, defined in section 4.
- **M3inv** is the GEI_{J2000} to SR1 transformation matrix. This is the inverse of the SR1 to GEI_{J2000} matrix, defined in section 3.

The above given matrix multiplication rotates the unit vector giving the Sun direction, as defined in the GSE system, successively to the GEI system, then to the GEI_{J2000} system, and then to the SR1 system.

Once the $\mathbf{T_{xyz}}$ = {x,y,z} vector is calculated, the angle θ is then given by the formulas:

$$\theta = \arccos\frac{x}{\sqrt{x^2 + y^2}} \qquad \text{if } y > 0$$

$$\theta = 360° - \arccos\frac{x}{\sqrt{x^2 + y^2}} \qquad \text{if } y \le 0$$

3. SR1 to GEI_{J2000}

As deduced from Figure 2.1, the transformation matrix $\mathbf{M_3}$ from the SR1 to the GEI_{J2000} system can be calculated as:

$$M3 := \begin{bmatrix} \cos\left(\frac{\pi}{2} - SPRASC\right) & \sin\left(\frac{\pi}{2} - SPRASC\right) & 0 \\ -\sin\left(\frac{\pi}{2} - SPRASC\right) & \cos\left(\frac{\pi}{2} - SPRASC\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(\frac{\pi}{2} - SPDECL\right) & \sin\left(\frac{\pi}{2} - SPDECL\right) \\ 0 & -\sin\left(\frac{\pi}{2} - SPDECL\right) & \cos\left(\frac{\pi}{2} - SPDECL\right) \end{bmatrix}$$

where:

- `SPRASC` is the right ascension of the spacecraft maximum principal inertia axis, in the system GEI_{J2000}, and is given in the spacecraft attitude file (`SATT`). The expected value for **`SPRASC`** is about **103°**.
- `SPDECL` is the declination of the spacecraft maximum principal inertia axis, in the system GEI_{J2000}, and is given in the spacecraft attitude file (`SATT`). The expected value for **`SPDECL`** is about **-64°**.

The negative `SPDECL` value is due to the southward pointing spin-axis.

For a spin axis pointing exactly to the South Ecliptic Pole, these values would be: `SPRASC` = 90.0°, `SPDECL` = -66.55°

4. GEI_{J2000} to GEI

The GEI_{J2000} to GEI transformation matrix **P**, or $\mathbf{M_4}$, gives the rotation from the GEI system, corresponding to year 2000, to the GEI system corresponding to the date the data were acquired ("mean of date" inertial system). **P** is also called the precession matrix, because it gives the precession of equinoxes.

The **P** matrix is calculated using the Fortran subroutine, given in *Ref. 1*, Appendix H.6. The subroutine code is also given in the ESOC Cluster CD-ROM.

For data acquired near year 2000 the **P** matrix is very close to the unit matrix. This transformation can thus simply be substituted by the unit matrix.

5. GEI to GSE

5.a Previous method (versions up to 5.0), less precise

The GEI to GSE (Geocentric Solar Ecliptic) transformation matrix, given in this section, is described in *Ref. 5*.

In the GEI system the unit vector in the direction of the Z_{GSE} axis, which is the direction of the ecliptic pole, is a known constant: **E** = {0, -0.3979, 0.9174} = $\{E_1, E_2, E_3\}$.

The direction of the X_{GSE} axis in the GEI system, which is the direction **S** of the Sun, can be computed from the `CSUNDI` subroutine (*Ref. 5, 6*). This subroutine gives the Sun right ascension `sra` and declination `sdec`. The Sun direction unit vector, in the GEI system, is then **S** = {cos (`sdec`)·cos(`sra`), cos(`sdec`)·sin(`sra`), sin(`sedc`)} = $\{S_1, S_2, S_3\}$ (Fig. 5.1).

The remaining row **Y** of the GEI to GSE transformation matrix, $\mathbf{M_5}$, is given by the relation:

$$Y1 := E2 \cdot S3 - E3 \cdot S2$$

$$Y2 := E3 \cdot S1 - E1 \cdot S3$$

$$Y3 := E1 \cdot S2 - E2 \cdot S1$$

The $\mathbf{M_5}$ (GEI to GSE) transformation matrix thus is:

$$M5 := \begin{pmatrix} S1 & S2 & S3 \\ Y1 & Y2 & Y3 \\ E1 & E2 & E3 \end{pmatrix}$$

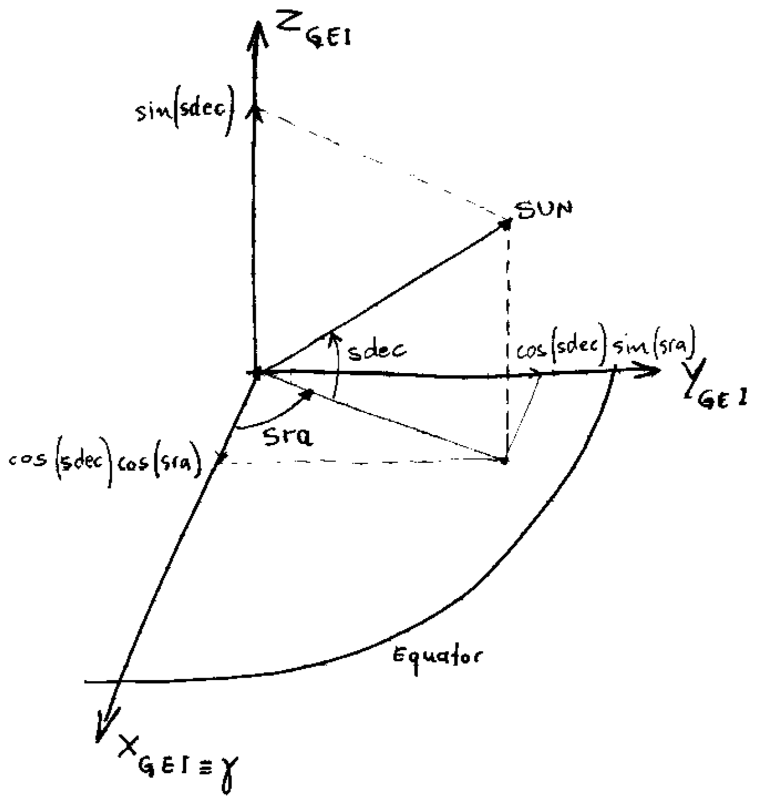

Fig. 5.1

Sun coordinates with respect to the GEI system: right ascension and declination

5.b New method (version 6.0), more precise

The GEI to GSE (Geocentric Solar Ecliptic) transformation matrix, given in this section, is described in *Ref. 7*.

The direction of the Sun, along the Ecliptic, is given by its longitude `L_sun` (Fig. 5.2). From spherical trigonometry, cos(`L_sun`) = cos (`sdec`) · cos(`sra`), so:

$$\texttt{L\_sun} = \arccos(\cos(\texttt{sdec}) \cdot \cos(\texttt{sra})) \qquad \text{if} \qquad \texttt{sra} < \pi$$

$$\texttt{L\_sun} = 2\pi - \arccos(\cos(\texttt{sdec}) \cdot \cos(\texttt{sra})) \qquad \text{else}$$

where `sra` and `sdec` are the Sun right ascension and declination. They can be calculated by using the `CSUNDI` subroutine, (*Ref. 5, 6*), or the `SUN` subroutine, (*Ref. 6, 7*).

The transformation of the components of a vector from the GEI system into the GSE system is then obtained by the product of two transformations: a rotation around the X_{GEI} axis by `obliq` (`obliq` is the obliquity of the Ecliptic), then a rotation around the Z_{GSE} axis by an angle `L_sun`:

$$\mathbf{M_5 = M_{5L} \bullet M_{5E}}$$

where:

$$M5E := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(obliq) & \sin(obliq) \\ 0 & -\sin(obliq) & \cos(obliq) \end{pmatrix} \qquad M5L := \begin{pmatrix} \cos(L\_sun) & \sin(L\_sun) & 0 \\ -\sin(L\_sun) & \cos(L\_sun) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The obliquity of the Ecliptic is approximately 23.43928°, but it can be calculated more accurately (as a function of the date) by using the `CSUNDI` subroutine, (*Ref. 5, 6*), or the `SUN` subroutine, (*Ref. 6, 7*).

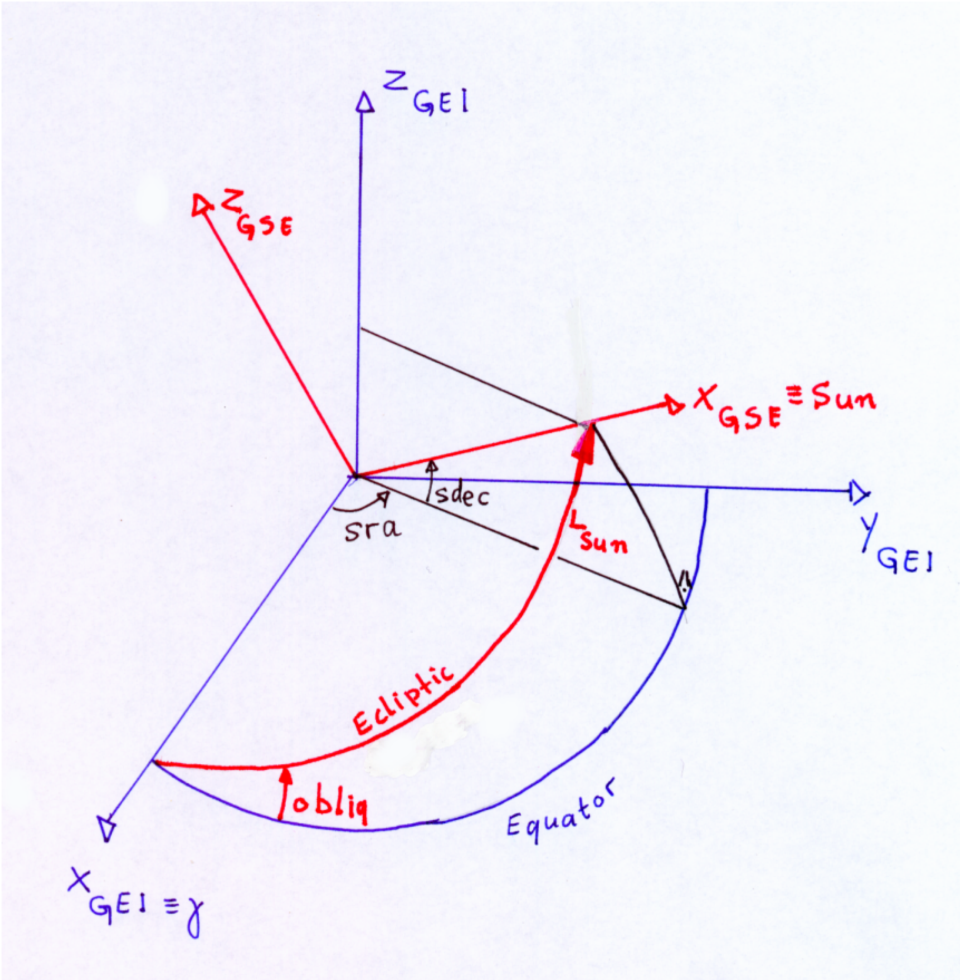

Fig. 5.2

Sun coordinates with respect to the GEI system: right ascension and declination