January 29, 1979 Library: TESTLIB

Subroutine: CPINT(BARR,WPAR,WPRP,FVALUE)

Author: V. M. Vasyliunas, documented by J. Belcher.

Purpose: Given a speed and pitch angle by PMODEL, to set up the phase angle integration about the magnetic field. The integration is actually done by PHINT. PMODEL provides WPAR ($W\cos\theta$), WPRP ($W\sin\theta$), and FVALUE. FVALUE is a product of the distribution function (which depends only on W and θ) and various velocity space volume elements. The currents for this ϕ integration are added to the current matrix BARR, as defined in PMODEL.

Method: CPINT has a DO loop whose index I runs from 1 to 5. The first four loops are for A,B,C, and D in the positive ion mode, and the fifth is for the D cup in the electron mode. We discuss the procedure for any one of those cups and modes. All vectors discussed below are in cup coordinates for that cup (see SETCPI). From the construction in SETCPI, it is clear that we are integrating over velocity vectors $\vec{v}$ specified by

$$\vec{v}(\phi) = \vec{V} + W_{\parallel}\hat{b} + W_{\perp}\cos\phi\hat{W}_{cs} + W_{\perp}\sin\phi\hat{W}_{sn} \qquad (1)$$

where $\vec{V}$ and $\hat{b}$ are the bulk velocity and magnetic field direction, and $\hat{W}_{cs}$ and $\hat{W}_{sn}$ are as defined in SETCPI. As ϕ ranges from 0° to 360°, we sweep out the velocity vectors of interest. Since $\hat{W}_{cs}$ has no Z - component in cup coordinates, we have a simple expression for the velocity v_z into the cup.

$$v_z(\phi) = V_z + W_{\parallel}b_z + W_{\perp}\sin\phi\, W_{sn,z} \qquad (2)$$

Thus, the maximum and minimum values of v_z over the range of ϕ are

$$v_z^{\max}_{\min} = V_z + W_{\parallel}b_z \pm |W_{\perp}W_{sn,z}| \qquad (3)$$

Remember that positive v_z is into the cup. If v_z^{max}

is less than zero, then no particle in this range enters the cup, and we return. If $v_z^{max} > 0$. the matrix UU is set to be either UUP (for $I \leq 4$) or UUE (for $I = 5$), as defined in equations (1) and (2) of the documentation for SETCPI. That is, UU is Z times the energy levels the particle will see as it moves into the cup, with the following sign convention. If $Z > 0$ and we are in a positive ion mode in A,B,C, or D, then UU will be positive. Similarly, if $Z < 0$ and we are in an electron mode in the D cup, then UU will be positive. However, if $Z > 0$ and we are in an electron mode in the D cup, then UU is negative. Similarly, if $Z < 0$ and we are in a proton mode in the D cup, or any mode in A,B, or C, then UU is also negative. In short, positive UU signifies electrons in an electron measuring mode, or protons in a proton measuring mode, and negative UU signifies protons in an electron measuring mode, or vice versa.

First consider the case that the voltages UU(J) are positive, J = 1,2...,NC (the number of windows, i.e., either 17 or 129). Then U(J) is set to the corresponding velocity (i.e., either UP or UE, as defined by SETCPI). In the simplest case, suppose there exist velocity channels such that

$$U(J_{low}) < v_z^{min} < U(J_{low}+1)$$

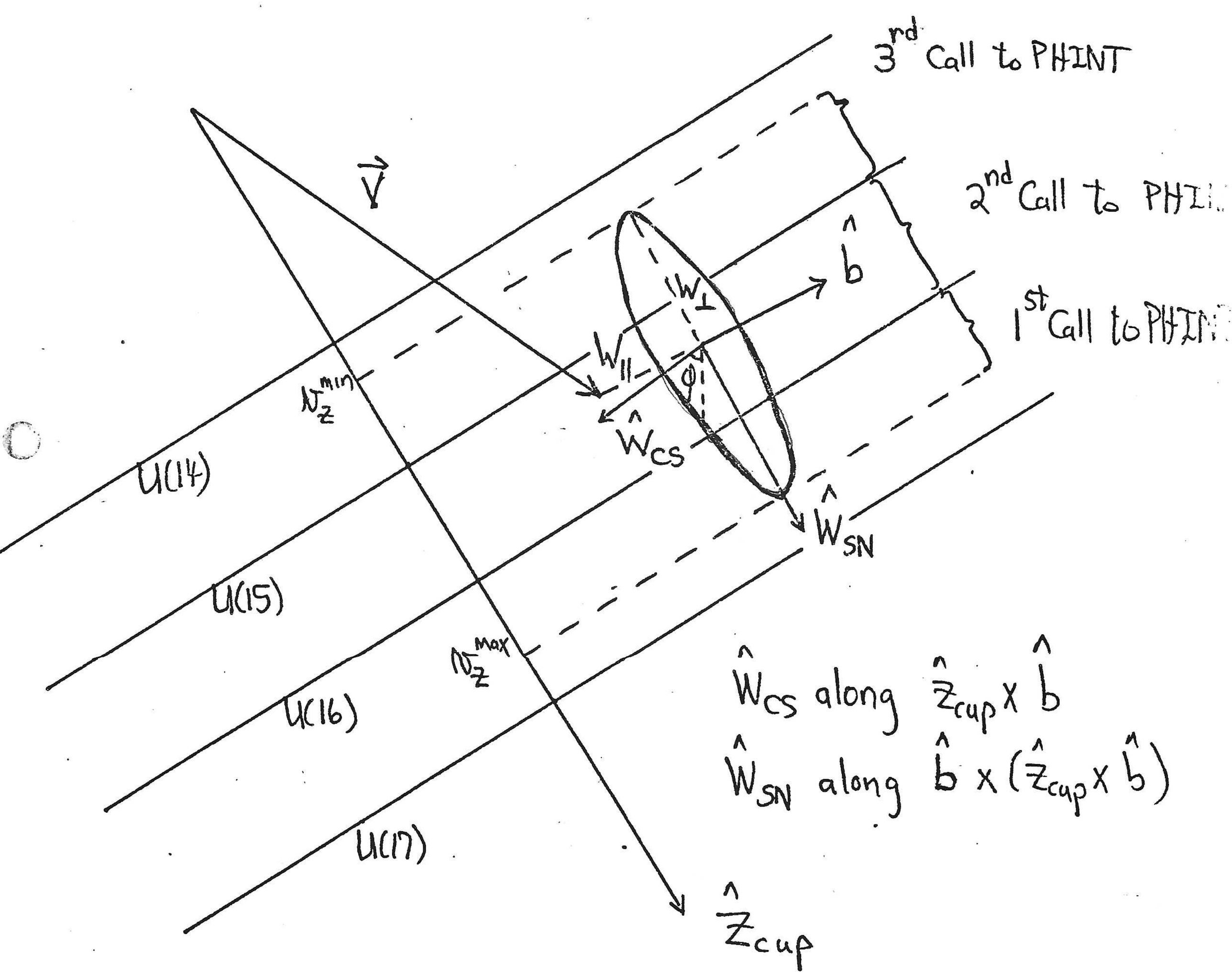
$$U(J_{hi}) > v_z^{max} > U(J_{hi}-1)$$

Then the routine divides the ϕ-integration into intervals as illustrated in Figure 1, i.e., the ϕ integration is divided into intervals such that the corresponding v_z is exactly at the velocity window. In the case illustrated, this is $J_{hi}-J_{low}$ intervals, with a corresponding number of calls to PHINT. For

each of these intervals, PHINT is called with the index of the lower energy window for that interval (i.e., 16, 15, and 14 in Figure 1), and the maximum and minimum values of sin ϕ for that interval. The values of sin ϕ at the velocity windows are easily found by setting v_z to the velocity window desired and inverting equation (2). The actual integration for a given interval is then carried out by PHINT, as described in the documentation for that routine. Note that J_{hi} may be $J_{low}+1$, in which case there is only one call to PHINT, with $(\sin\phi)_{min} = -1$ and $(\sin\theta)_{max} = +1$. Similarly, v_z^{min} may be greater than the highest velocity window, so that again there is only one call to PHINT with the same limits. It is also possible that v_z^{min} is lower than the lowest velocity window, in which case the integration is cut off at that value of sin θ such that v_z equals the lowest window. Similarly, v_z^{max} may be lower than the lowest velocity window, in which case we return with no call to PHINT.

Now consider the case in which UU < 0 (i.e., electrons in a proton mode or protons in an electron mode). If $v_z^{min} > 0$, all particles enter the cup, and we call PHINT for the full integration ($\sin\theta_{min} = -1$ and $\sin\theta_{max} = +1$). If $v_z^{min} < 0$, then we call PHINT with $\sin\theta_{max} = +1$ and $\sin\theta_{min}$ such that $v_z = 0$. In either case, we also provide to PHINT the index of the highest channel, i.e., either 17 or 129, and PHINT then carries out the θ-integration.

Subroutines called: PHINT



$\hat{W}_{CS}$ along $\hat{z}_{cup} \times \hat{b}$

$\hat{W}_{SN}$ along $\hat{b} \times (\hat{z}_{cup} \times \hat{b})$

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Library: TESTLIB

Subroutine: PHINT(BARRI,SNMAX,SNMIN,UU,KKK,WPAR,WPRP,III)

Author: V. M. Vasyliunas, documented by J. W. Belcher

Purpose: Carries out the ϕ-integration set by CPINT, with differencing of adjacent channels to produce the AC modulation amplitude which would be observed. No differencing of the highest channel (17 or 129), so as to obtain a "DC return" estimate.

Method: Resulting "currents" are stored in BARRI. SNMAX and SNMIN are the sin of ϕ_{max} and the sin of ϕ_{min}, respectively, as defined in the documentation for CPINT. The quantities UU are the energy levels as defined in CPINT, with KKK the index of the lower velocity window if UU > 0 (see CPINT). The quantities WPAR and WPRP are defined in CPINT and III is the index I defined in CPINT.

For each range of ϕ, we divide the interval ϕ_{min} to ϕ_{max} into subintervals $\Delta\phi$ of size

$$\Delta\phi = (\phi_{max} - \phi_{min}) * \frac{\varepsilon VV}{WPRP} \qquad (1)$$

where

$$VV = |\vec{V} + W_{\parallel}\hat{b} + W_{\perp}\sin\phi_{max}\hat{W}_{sn}| \qquad (2)$$

and ε is set in the common block ACCU. We then generate vectors along this range of ϕ, at these intervals. For each vector, the routine CPRESP (cup response) is called, <u>for every energy level from UU(KKK) down to UU(1)</u>. That is, we calculate the contribution of each vector in this range not only to KKK, but to every channel below KKK, down to the first. Differencing those contributions <u>below</u> KKK provides the amplitude of the AC modulation due to refraction effects. If UU > 0 and KKK is not the top window, then the undifferenced contribution at

KKK is the amplitude of the AC modulation itself, since we are guaranteed that no particles in this range of ϕ lie above the KKK+1 window. If UU > 0 and KKK is the upper energy window, the contribution at KKK is the DC return. If UU < 0, then KKK must be the top of the range, and the contribution at KKK is the DC return, with the differenced contributions below KKK the AC modulation due to refraction effects.

Subroutines Called: CPRESP

January 29, 1979

Subroutine: PMODEL (BARR,V,B,Z,A,MP,ME,DSTFCN,INT,/W1/,/W2/,/DW/

Author: V. M. Vasyliunas, documented by J. W. Belcher

Purpose: Computes currents produced in the Voyager plasma experiment for a given species, given the species bulk velocity V, the field direction B, and any gyrotropic distribution function.

Usage: Given either an ion or electron distribution function, we wish to compute the response of the Voyager detectors (main cluster and side sensor) to that particle population, in any mode (E1,E2,L, or M). This includes proton contributions to the electron mode (i.e., due to refraction/modulation effects), and vice versa. On a given call to PMODEL, we specify: the bulk velocity of the species, V(3); the field direction, B(3); the charge number of the particle, Z (+1 for protons, -1 for electrons); the mass of the particle in AMU, A (if A < 0.9, the electron mass 1/1836 is used). We also specify the name of a function subroutine DSTFCN (W,CS), which returns a value of the distribution function for this (Z,A) species at the speed W and cosine of the pitch angle CS, as measured in the rest frame of the species. The units of V and W, and all velocities used subsequently, are cm/sec. The distribution function should have units of $sec^3\ cm^{-6}$. The currents returned by the program are in femtoamps (10^{-15}). The units of B are arbitrary. All directions are specified in spacecraft XYZ coordinates.

For the given species, MP determines the ion mode for which we are calculating currents: MP = 0 is the L mode; MP = 1 is the M mode. The parameter ME determines the electron mode: ME = 0 is E2; ME = 1 is the E1 mode. The resulting currents are stored in BARR (129,5). The first index is the channel number. For the M mode, 1 → 128 contains the differential currents for the 128 channels, and 129 contains the integral current above the highest energy window. For the other modes, 1 → 16 contains the differential currents, and 17 contains the integral current above the highest energy window. The second index of BARR defines the cup and mode: 1,2,3,4 are the A,B,C, and D cups in the positive ion mode (either L or M, as specified by MP); 5 is the D cup in the electron mode (either E1 or E2, as specified by ME).

For example, if PMODEL is called with (Z,A,MP,ME) = (1.,1.,0,0), BARR(I,1) for I = 1,2...,17 would contain currents for the A cup in the L mode for a proton particle population. BARR(I,4) for I = 1,2,...,17 would contain currents for the D cup in the L mode for a proton particle population. BARR(I,5) for I = 1,2 ...,17 would contain the contributions of this proton population to the D cup in the E2 mode (i.e., due to refraction/modulation effects). If we want the contributions of this proton population to the D cup in the E1 mode, we must call PMODEL a second time with ME = 1.

As a second example, if PMODEL is called with (Z,A,MP,ME) = (-1.,0.,0,1), BARR(I,J) for J = 1,2,3,4 would contain the contributions of an electron population to the A,B,C, and D cups in the L mode. BARR(I,5) would contain the contributions of an electron population to the D cup in the E1 mode.

The differential currents in BARR are positive for the expected AC phase, in both positive ion and electron modes (i.e., electron population in E1 mode produces positive currents, as does proton population in L mode). If the differential currents are not in the AC phase expected, they appear as negative. In the integral channel, protons in the electron mode will appear as negative currents, and vice versa.

Note that BARR is not cleared by PMODEL; the results are simply added to what is already in BARR. The meaning of INT, W1, W2, and DW are explained below.

Problem Description: For a given species (Z,A) with bulk velocity V and distribution function in the rest frame DSTFCN (W,CS), we must convolve the particle population with the response of the instrument to that species for a given cup, mode, and channel number. This involves an integration over the species' speed W, pitch angle θ, and phase angle ϕ about the field, multiplied by the value of the distribution function, the appropriate phase space differential volume, and the response of the instrument for a given cup, mode, and channel.

Method: I. The Default Integration Scheme (INT = 0)

The routine PMODEL first calls a subroutine SETCPI which sets up the energy and velocity windows for the desired modes, and constructs cup coordinates which are convenient for performing the integrations. In

particular, $\hat{Z}_{cup}$ is defined to be <u>opposite</u> the direction of the cup normal $\hat{n}_{cup}$, and VCP (3,I) for I = 1,2,3,4 is the component of the bulk velocity $\vec{V}$ along $\hat{Z}_{cup}$ for the A,B,C, and D cups, respectively. Note that with this convention, if $\vec{V}$ is directly <u>into</u> the A cup, VCP (3,I) will be <u>positive</u> with a value $|\vec{V}|$. See SETPHI for details.

PMODEL then proceeds to set up the integration over particle speed W and pitch angle θ with respect to the field. We first describe the default integration scheme (INT = 0, W1, W2 and DW are not used and do not have to appear in the calling sequence). The speed integration is carried out from 0 to essentially infinity (see below), with zero speed defined as motion at the bulk velocity $\vec{V}$. For each value of the speed, PMODEL sets up an integration over the cosine of pitch angle θ. The routine CPINT, for a given speed and pitch angle, then carries out the phase angle (ϕ) integration (see Figure 1). Quantitatively, we need to carry out an integral of the form

$$\iiint [\ \ldots] \ W^2 \, dW \, d(\cos\theta) \, d\phi \qquad (1)$$

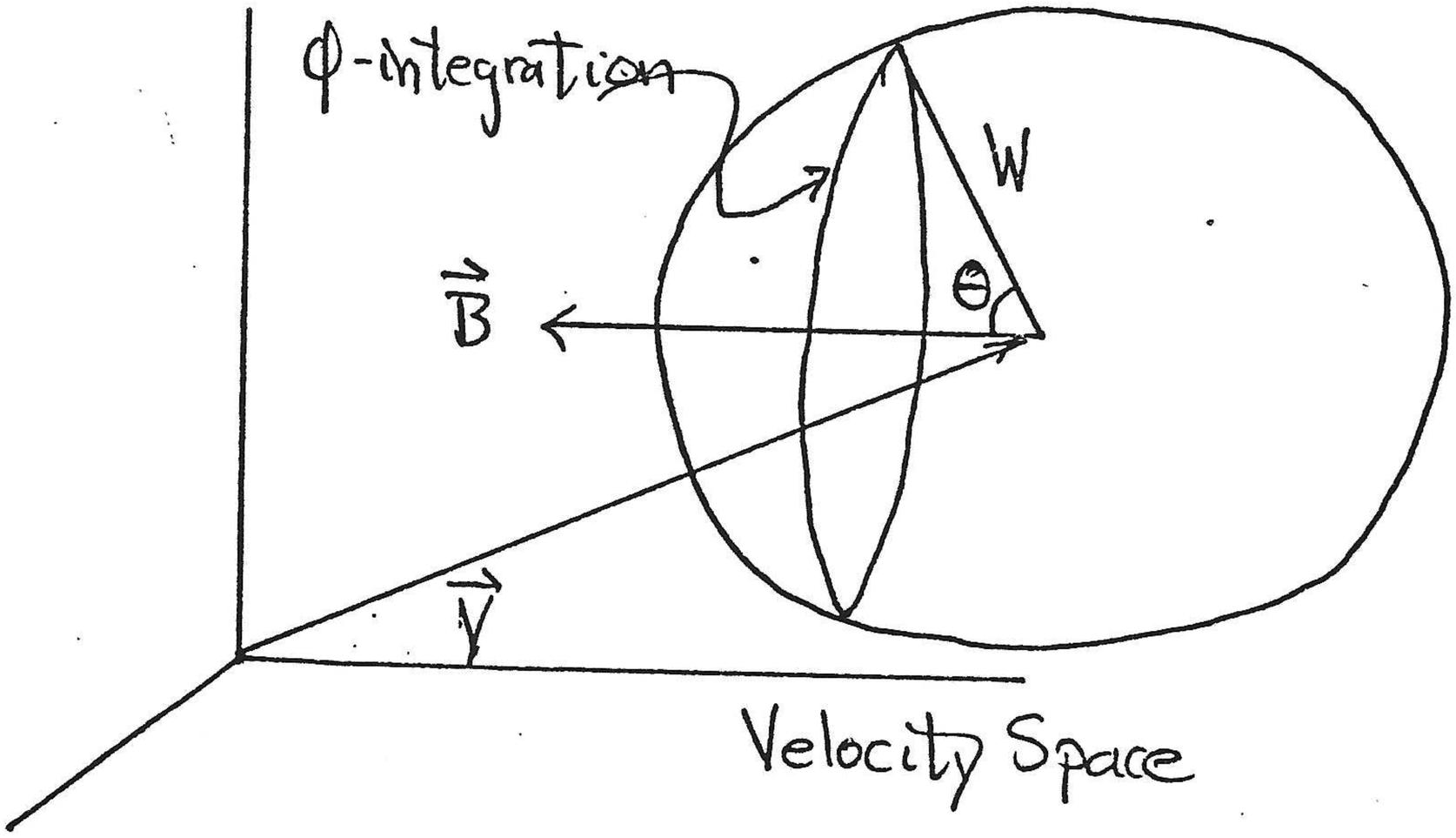


FIGURE 1

where [...] includes, among others, the distribution and response functions. PMODEL sets up the W and θ integrations, and CPINT sets up the ϕ integration.

The default integration (INT = 0) in PMODEL proceeds as follows. The integration in W is linear from 0 to a value WL, with step size DU. It is log in W from WL to a value WI, and then inverse in W from WI to essentially infinity. These limits are determined as follows.

A. The Linear Portion of the Speed Integration ($0<W\leq$ WL)

First, consider positive ions ($Z\geq 1.$). For the appropriate ion mode (L or M, depending on MP), SETCPI has calculated the velocities associated with each of the energy levels of that mode, for that species (Z,A). If U_{min} is the velocity at the lowest energy window, then

$$WL = AMAX[2.5*U_{min}, |VCP(3,I)-U_{min}|_{I = A,B,C,D}] \quad (2)$$

This, insures that the linear portion of the speed integration will extend from 0 in the rest frame to a value which for some θ and ϕ is at, or less than, the lowest energy level in the spacecraft frame, for all cups. DU is initially set to one-eighth of the difference between the second velocity from the lowest and the lowest velocity. It is then adjusted upward fractionally so that WL is an exact integral multiple of DU.

For electrons ($Z<0.$), SETCPI has calculated the velocities associated with each of the energy levels of E1 or E2, depending on ME. If U_{min} is the velocity at the lowest energy window, then

$$WL = AMAX[2.5*U_{min}, |VCP(3,4)-U_{min}|] \quad (3)$$

The value of DU is set in the same way as above.

Thus, for either positive ions or electrons, in the linear portion of the integration, the speed at the J-th step is

$$W_J \quad J*DU,\ J = 1,2,\ \ldots NSTEP,$$

where NSTEP*DU = WL. At the J-th step in the W integration, the cos θ integration, extending from -1 to 1, is divided up into $2W/\Delta W$ intervals, so that the step size is cos θ is 1/J. As W_J increases, the number of cos θ intervals increases also, such that the step size in $W_{||}$ for a given W is constant at DU, since $W_{||}$ is W cos θ.

B. The Log Portion of the Speed Integration of (WL$\leq$W<WI)

For the default integration, WI is set to twice the maximum velocity associated with highest energy level in the proton mode, if we are dealing with protons, or in the electron mode, if we are dealing with electrons. We start at a value of W equal to WL + DU. For the J-th step in this regime,

$$W_J = (WL + DU)\exp\left[\frac{DU}{WL}(J-1)\right] \qquad (4)$$

for J = 1,2,...NSTEP, where NSTEP is such that W_{NSTEP} has just exceeded WI as defined above. This gives velocity steps such that $\Delta W/W \simeq DU/WL$.

For the cos θ integration in this range of speeds, the number of intervals in η is again approximately $2W/\Delta W$, as above. In the log portion, the number of intervals in cos θ is therefore essentially constant at the number at the top step of the linear integration, e.g., 2*WL/DU. Thus, the step size in $W_{||}$ in this regime increases from DU linearly as W (i.e., $\Delta W_{||} = DU(W/WL)$).

C. The 1/W Portion of the Speed Integration (W_I<W<∞)

Once we reach the value W_{NSTEP} defined above, we redefine W_I slightly as exactly W_{NSTEP}, and proceed with the 1/W integration. For the J-th step in this regime,

$$W_J = [\frac{1}{WI} - DD*(J-1)]^{-1}$$

where

$$DD = (\exp(DU/WL)-1)/WI$$

or

$$DD \simeq \frac{DU}{WL} \quad \frac{1}{WI}$$

Thus, in 1/W space, we are breaking up the internal (0,1/WI) into WL/DU steps. The integration is stopped at the smallest value of W_J that is still positive. This is a value of W that exceeds 1/DD, or $\simeq$ WI(WL/DU). This is the definition of "infinity".

For the $\cos\theta$ integration, the number of steps is as before proportional to $2W/\Delta W$. In this regime,

$$\frac{2W}{\Delta W} \simeq 2 \frac{WL}{DU} \quad \frac{WI}{W}$$

At the lower end of the range (W_I), this is $\simeq 2 \frac{WL}{DU}$, as above. At the upper end (WI*WL/DU), this is $\simeq 2$. The number of $\cos\theta$ intervals decreases as W^{-1} in this regime.

II. Integration Scheme for INT = 1.

For INT = 1, we must have W1 and W2, although DW is not used and does not have to be specified in the calling sequence. The linear/log/inverse integration regimes are as described above, with a change in definition of WL and WI. If W1 is less than or equal to zero, then WL is defined as above. If W1 is greater than zero, then WL is the lesser of WI and the value computed above (cf. equation(2)). If W2 is less than or equal to zero, then WI is defined as above. If W2 is greater than zero, then

WI is the lesser of .W2 and the value defined above (twice the velocity at the highest energy level).

Basically, INT = 1 provides the flexibility of varying the linear/log/inverse boundaries away from the default values towards lower limits. If WI turns out to be less than WL (i.e., by choice), then the log portion of the integration is not done, and the integration is inverse above WL. In any case, the variable DU is <u>not</u> changed from the above choice.

III. Integration Scheme for INT = 2

Routine performs a linear integration between W1 and W2 <u>only</u>, using step size DW. The number of intervals for the $\cos\theta$ integration is as before $2\ W/\Delta W$, where ΔW is now DW. This option allows integration over distributions which are "thin" in some sense - e.g., Siscoe's satellite ion distributions.

Additional Notes on Running PMODEL

For debugging purposes, and for more flexibility in running PMODEL, the following additions have been made to the three entries in COMMON/ACCU/. These entries are all set in BLOCK DATA so that the program runs in its nominal mode (i.e., no diagnostic output and the nominal step size DU, etc.). These parameters are as follows. If the integer IBG1 is non-zero, the matrix BARR is written out at the end of each step in the W integration, on unit IBG1. In addition, for cup and mode IIIBG1 (running from 1 to 5, as described for the variable I in CPINT), a number of parameters are written out for every call to PHINT for that cup and mode again on unit IBG1. This information includes the speed and pitch angle for that call to PHINT, the maximum and minimum of sin ϕ for that call, the RANGE in degrees for the integration over ϕ, the number of steps NSTEP to be used in the ϕ-integration, the lower energy channel KKK, the value of IIIBGI, and the parameter VV. In addition, if IPHBG is non-zero, for every 50-th ϕ -integration, we write out information for every step in the ϕ -integration, including the number N of the step, the vectors used in the integration, the variable SAVE, which is the return from CPRESP, the relevant channel voltage, the suppressor voltage, ZCU, and AEVV.

If IBG2 is non-zero, we proceed exactly as above for cup and mode IIIBG2, with the output appearing on unit IBG2. If IBG2 is non-zero, IBG1 must also be non-zero.

If NDOCUP(I), I = 1,2,3,4,5 is non-zero, then for cup and mode I, as described in CPINT, no integration will be done. For example, if the user does not want to calculate the proton contribution to the electron mode, NDOCUP(5) should be set to one, the others left at zero.

The basic integration step DU is the difference between the 1st and 2nd step velocities divided by the parameter STPSZ. The default value is 8.0, as described above, but this may be changed

to smaller values to decrease the total number of steps.

For each ϕ -integration range, there is a lower energy window KKK. The nominal logic computes the cup response for every step from KKK to one, differencing those below KKK. We now stop that process at a step K such that the difference between the response K and the response K + 1, divided by response K, is less than EPSMOD in absolute value. The default value of EPSMOD is 0.0, which is the nominal run.

The common block now appears as follows:

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COMMON/ACCU/EPS,PVSTP,EVSTP,IBG1,IIIBGI,IB2,IIIB2,IPHBG,NDOCUP(5),
   STPSZ,EPSMOD
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Subroutine: SETCPI(V,B,Z,A,MP,ME)

Author: V. M. Vasyliunas, documented by J. W. Belcher

Purpose: To set up energy levels and coordinate systems for the Voyager simulation program. The results are accessed through the common block CPI.

Problem Description: For a given species (Z,A) and cup mode (L,M,E1 or E2), we need to set up energy levels and velocities at those energy levels in a manner which is appropriate to the species. We also need to set up cup coordinates which facilitate the $W\Theta\phi$ integrations described in PMODEL.

Method and Usage: Definition of the arguments in the calling sequence are as in PMODEL. We define in order the elements of the common block CPI.

Cup coordinates are defined as follows. The $\hat{Z}_{cup}$ axis is anti-parallel to the cup normal (i.e., it points <u>into</u> the cup). The $\hat{X}_{cup}$ direction is along $\hat{Z}_{cup} \times \hat{Z}_{sc}$, where $\hat{Z}_{sc}$ is the spacecraft Z-axis. The $\hat{Y}_{cup}$ direction completes the right-handed system. The matrix TM(9,4) in SETCPI is the rotation matrix <u>from</u> s/c coordinates <u>to</u> cup coordinates so defined, for each of the four cups.

The matrix VCP(3,I) is then the bulk velocity V as seen in each of the four cup coordinate systems, I = 1,2,3,4 corresponding to A,B,C,D. Similarly, for BCP(3,I), except that BCP is normalized to a unit vector. The matrix WCS(3,I) is the unit vector along $\hat{Z}_{cup} \times \hat{B}_{cup}$, for each of the four cups; WSN(3,I) is the unit vector along $\hat{B}_{cup} \times (\hat{Z}_{cup} \times \hat{B}_{cup})$.

For a given cup, the triad of vectors $(\hat{W}_{cs}, \hat{W}_{sn}, \hat{B}_{cup})$ form a right-handed system, and it is this system which is most convenient for doing the $W\Theta\phi$ integration described in PMODEL. The phase angle ϕ is measured from $\hat{W}_{cs}$ positive toward $\hat{W}_{sn}$, as usual, with Θ measured from $\hat{B}_{cup}$. Note that $\hat{W}_{cs}$ has no component along $\hat{Z}_{cup}$, a fact that will be exploited repeatedly .

The matrices UUP and UUE are essentially the energy levels for the positive ion mode (L or M, depending on the value of MP) and of the electron mode (E1 or E2, depending on the value of ME), with important differences. In particular,

$$UUP(I) = Z[60\ \varepsilon^{I-1} - 50] \qquad (1)$$

where ε depends on whether we are in the L or M mode. Note that the normal voltage is multiplied by Z, and has the sign of Z. It is negative if $Z < 0$. For electrons,

$$UUE(I) = -Z\ [60\ \varepsilon^{I-1} - 50] \qquad (2)$$

where ε depends on whether we are in E1 or E2. Note that UUE has the sign of -Z, and is positive only if $Z < 0$.

The matrices UP and UE are velocities at these energies appropriate to the species. If $Z > 0$, UP is $[(2e/Am_p)*UUP]^{1/2}$, where e is the electron charge, m_p the proton mass, and the units produce cm/sec. In this case, UE is not set. If $Z < 0$ and $A < 0.9$, UP is not set, and UE is $[(2e/m_e)*UUE]^{1/2}$, in cm/sec (m_e is the electron mass). If $Z < 0$ and $A \geq 0.9$, UP is not set, and UE is $[(2e/Am_p)*UUE]^{1/2}$. Note that this allows us to handle negative ions.

If MP = 1, NCP is 129, and if MP = 0, NCP is 17. The parameter NCE is always 17. The parameter SUPMN is -95*Z (i.e., the effective suppressor voltage in the main sensor for the charge Z). The parameter SUPSD is -95Z if the D cup is in the E2 mode (ME = 0) and approximately -8Z if the D cup is in the E1 mode (ME = 1). The parameter ZCU is Ze, with e measured in femtocoulombs ($e = 1.6 \times 10^{-4}$ femtocoulombs). AEVV is $Am_p/2e$ if $A > 0.9$ and $m_e/2e$ if $A < 0.9$, with units so that AEVV times a voltage yields $(cm/sec)^2$.

Subroutines called: None